Title: Summary/Review of Hypothesis Testing/Confidence Intervals

Class: Math 117

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Instructions to tutor: Carefully read and follow instructions of the worksheet presented by the student. Do not deviate from these instructions or teach any "short-cuts." If the student has a good grasp of the material, solutions have been provided for you, but not the student, to help you move quickly through the worksheet.

Objective: To review many different types of hypothesis testing and confidence intervals. To confront the student with many different types of problems, all on the same page, prior to an Exam. To emphasize the importance of being able to categorize problems before you begin to work them out.

Activity: See "Student Direction' on page one of the student hand-out.

Summary of Hypothesis Testing/Confidence Intervals

Student Directions:

1. Go through your textbook/lecture notes and identify every hypothesis test or confidence interval we have encountered this semester.
2. Check with a tutor or your instructor to make sure that your list in part (1) is complete.
3. Carefully read **Practice Problems 1 – 8** (below) and decide which test or interval is appropriate. Do NOT begin to test! Instead, only categorize!!
4. Return to your tutor and discuss your choice of test or interval for each problem. Make sure the tutor agrees with you on every problem before you invest a lot of time into reaching a conclusion or creating an interval.
5. Pick one or two problems and go through the structured steps discussed in class and outlined below to complete the hypothesis testing or to create the interval. Write out your solutions on a separate piece of paper.
6. Then try at least two problems on your own.
7. Return to the tutor to see how successful you were in carrying out the test or finding the interval.
8. On a separate sheet of paper, finish the problems and then check the correctness with the tutor.
9. When you are satisfied with your conclusions/intervals, submit all work and the signed tutor form.

Review of steps for creating a confidence interval or conducting a hypothesis test:

- Do not deviate from the steps below.
- Use a calculator for computations and calculating the area under a curve, but do not use the testing of interval features. Instead, calculate by following the steps below.

Confidence Interval: OBS $\pm$ Margin of Error	**Hypothesis Testing**
1) Find observation, t* or z*, and spread 2) Plug it all in to formula and show all work. 3) Write it as an interval: (lower bound, upper bound) 4) Conclude that you are confident that any value inside the interval is a reasonable "guess" for the true parameter (mean or probation) and anything outside the interval is unlikely.	1) State the hypothesis. 2) Calculate the test statistic 3) Plot and shade on appropriate distribution. Make sure to label the distribution! 4) Calculate p-value. 5) Make a decision. 7) Write your conclusion in the context of the problem.

Practice Problems:

1. According to official census figures, 8% of couples living together are not married. A researcher took a random sample of 400 couples and found that 9.5% of them are not married. Test at the 15% significance level if the current percentage of unmarried couples is different from 8%.

2. The mean weekly earnings of a sample of 30 construction workers was $759, with a standard deviation of $73, and the mean weekly earnings of a sample of 28 manufacturing workers was $658, with a standard deviation of $65.
   a) Construct a 80% confidence interval for the difference between the mean weekly earnings for construction workers and the mean weekly earnings for manufacturing workers. [choose appropriate value: $s_d = 45.60$ or $s_p = 69.26$]
   b) Explain what the confidence interval mean
   c) Is it reasonable to say that there is not a significant difference in pay between construction workers and factory workers? Carefully explain your reasoning.

3. According to a study, 21.1% of 507 female college students were on a diet at the time of the study.
   a) Construct a 99% confidence interval for the true proportion of all female students who were on a diet at the time of this study.
   b) Explain what this interval means.
   c) Is it reasonable to think that only 17% of college women are on a diet?

4. A psychologist claims that the mean age at which children start walking is 12.5 months. The following data give the age at which 18 randomly selected children started walking.

15	11	13	14	15	12	15	10	16
17	14	16	13	15	15	14	11	13

Test at the 1% level of significance if the mean age at which children start walking is different from 12.5 months.

5. A sample of 800 items produced on a new machine showed that 48 of them are defective. The factory will get rid the machine if the data indicates that the proportion of defective items is significantly more than 5%. At a significance level of 10% does the factory get rid of the machine or not?

6. A random sample of eight students was selected to test for the effectiveness of hypnosis on their academic performance. The following table gives the GPA for the semester before and the semester after they tried hypnosis.

Before	2.3	2.8	3.1	2.7	3.4	2.6	2.8	2.5
After	2.6	3.2	3.0	3.5	3.7	2.4	2.9	2.9

a) Construct a 99% confidence interval to see if hypnosis improves academic performance. [choose appropriate value: $s_d = 0.316$ or $s_p = 0.392$]
b) According to the interval, clearly state if the data supports that claim that the hypnosis changed the academic performance.
c) Test at the 1% significance level if there is a change in the academic performance of students due to hypnotism.

7. The population standard deviation for waiting times to be seated at a restaurant is know to be 10 minutes. An expensive restaurant claims that the average waiting time for dinner is approximately 1 hour, but we suspect that this claim is inflated to make the restaurant appear more exclusive and successful. A random sample of 30 customers yielded a sample average waiting time of 50 minutes.
   a) Is there evidence to say that the restaurant's claim is too high?
   b) The original data (individual waiting times) is not normally distributed. What theorem allows us to do the calculations in part (a)?

   State the title of the theorem and explain its content in your own words of picture.

8. In a survey of 1273 adults, 52% said it is not morally wrong to change the genetic makeup of human cells. What test would be indicated to address the following statement: "The majority of adults do not think it is morally wrong to change the genetic makeup of human cells"? Conduct this test.

For tutor use: Please check the appropriate box.

- ☐ Student has completed worksheet but may need further assistance. Recommend a follow-up with instructor.
- ☐ Student has mastered the material in this exam and understands/recognizes a pattern.

Tutor Name:______________________________

Tutor Signature:___________________________

Solutions to Summary of Hypothesis Testing/Confidence Intervals Chapter 8 – 11

Answers may vary due to rounding issues. I round to three places past the decimal AFTER I have done the major calculations. However, if you show your work, you may round during calculation if you would prefer.

1. Proportion z-test

$$H_0 : p = 0.08$$
$$H_1 : p \neq 0.08$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{.095 - .08}{\sqrt{\frac{.08(1-.08)}{400}}} = \frac{.015}{.013...} = 1.106$$

pvalue = 2normalcdf(1.106, 99999)=2(.134)=.269

Decision: Keep H_0. (because 26.9%>15%)

Conclusion: According to the observed data, the current percentage of couples living together who are unmarried is approximately equal to 8%.

2. Average, 2-sample Independent confidence interval

a) $$(\bar{x}_1 - \bar{x}_2) \pm t^* \left(s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right) = \begin{array}{l} (759 - 658) \pm 1.303\left(69.26\sqrt{\frac{1}{30} + \frac{1}{28}} \right) \\ 101 \pm 1.303(69.26 * 0.263) \\ 101 \pm 23.714 \\ (77.286, 124.714) \end{array}$$

Note: df = $n_1 + n_2 - 2$ = 56
but you must use "40" because "56" is not available. You shouldn't use "60" because that would be assuming a larger sample size. It is better to be conservative, and use the smaller sample size if your size is not available.

b) We are 80% confident that the true "average difference" between construction workers' weekly pay and factory workers' weekly pay is between $77.29 and $124.71.

c) No. If there were reasonable to think that there is no difference in pay, then our confidence interval would contain 0. The lowest point on our interval is $77.29, which mean that the lowest reasonable average difference in pay is more than $77.29.

3. Proportion confidence interval

a) $\hat{p} \pm z^* \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$ =

$$0.211 \pm 2.576\sqrt{\frac{0.211(1-0.211)}{507}}$$
$$0.211 \pm 2.576(0.018)$$
$$0.211 \pm 0.047$$
$$(0.164, 0.258)$$

b) We estimate the true percentage of female college students who are on a diet is somewhere between 16.4% and 25.8%. In fact, we are 99% confident that the true dieting percent is between those boundaries.

c) Yes, it is reasonable to estimate that 17% of female students are on a diet because 17% is in our confidence interval.

4. Average, one sample, σ unknown

$$H_0 : \mu = 12.5$$
$$H_1 : \mu \neq 12.5$$

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}} = t = \frac{13.833 - 12.5}{1.917/\sqrt{18}} = \frac{1.333}{0.452} = 2.950$$

pvalue = 2tcdf(2.950, 99999,17)=2(0.005)=0.009

Decision: Reject H_0. (because 0.9%<1.0%)

Conclusion: According to the observed data, the average age at which a child starts walking is actually different from the original claim of 12.5 months.

5. Proportion hypothesis test

$$H_0 : p = 0.05$$
$$H_1 : p > 0.05$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{.06 - .05}{\sqrt{\frac{.05(1-.05)}{800}}} = \frac{.01}{.0077...} = 1.300$$

value = normal(1.300, 99999)=0.096

Decision: Reject H_0. (because 9%<10%)

Conclusion: According to the observed data, the machine should be replaced because the proportion of defective items seems significantly higher than 5%.

6. Average, two-sample MATCHED PAIR confidence interval

a) $\bar{d} \pm t^{*}\left(\frac{s_D}{\sqrt{n}}\right)$ =

$$\begin{aligned} &.25 \pm 3.499\left(\frac{.316}{\sqrt{8}}\right) \\ &.25 \pm 3.499(.112) \\ &.25 \pm .391 \\ &(-0.141, 0.641) \end{aligned}$$

b) No, the data does not support the claim. Zero is the in the above interval. Therefore, it is possible that there is no real, significant difference between the overall average of the "before" and "after" test scores.

c)
$$H_0 : \mu_d = 0$$
$$H_1 : \mu_d \neq 0$$

$$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} = \frac{.25 - 0}{.316 / \sqrt{8}} = \frac{.25}{.112} = 2.238$$

pvalue = 2tcdf(2.238, 99999,7)=2(.030)=0.060

Decision: Reject H_0. (because 3%>1%)

Conclusion: The data indicates that hypnosis does not really change the academic performance of the students (just like we wrote in part1!!!)

7. Average, one-sample, σ known.

a)
$$H_0 : \mu = 60$$
$$H_1 : \mu < 60$$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{50 - 60}{10 / \sqrt{30}} = \frac{-10}{1.826} = -5.476$$

pvalue = normalcdf(-99999,-5.476)=(0.005)=0.0000000215

Decision: Reject H_0. (Because the pvalue is almost 0 – indicating that the data is very unlikely assuming H_0 is true.)

Conclusion: The restaurant is exaggerating the amount of time custromers have to wait to get a seat. In reality, it appears that the waiting time is less than an hour.

b) The Central Limit Theorem: If your original data is non-normally distributed with the center equal to μ and a population spread σ, then the distribution of $\bar{x}$ is approximately normal with the same center of μ and a population standard deviation of $\frac{\sigma}{\sqrt{n}}$. You can only be sure that the $\bar{x}$ distribution is approximately normal if the sample size n is big enough, $n \geq 30$ for most data sets.

8. Proportion hypothesis test

Note: "Majority" means "Greater than 50%"

$H_0: p = 0.50$

$H_1: p > 0.05$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{.52 - .5}{\sqrt{\frac{.5(1-.5)}{1273}}} = \frac{.02}{.014...} = 1.427$$

pvalue = normalcdf(1.427, 99999)=0.077

Decision: (Reject H_0 if $\alpha = .10$ or) keep H_0 if $\alpha = .05$. If α is not given, assume $\alpha = .05$.

Conclusion: ($\alpha = .05$)According to the observed data, about 50% of the people would say that it is not morally wrong to change the genetic makeup of human cells.