Title: Logarithmic Equations, Level II
Class: Math 111or Math 120 or Math 137
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Instructions to Tutor: Read instructions and follow all steps for each problem exactly as given.
Keywords/Tags: logarithmic equations, equations with logarithms, solving logarithmic equations, solving logarithm equations

Logarithmic Equations, Level II

Purpose: This is intended to refresh your skills in solving more complicated logarithmic equations.

Activity: Work through the following activity and examples. Do all of the practice problems before consulting with a tutor.

- **Definition:** for $b > 0,\ b \neq 1,\ \log_b a = x$ is equivalent to $b^x = a$.

  the answer to the logarithm is the exponent

  Note that the base *b* is a positive number, and that the number you are taking the logarithm of, *a*, is also a positive number. But, the answer to the logarithm, *x*, may be a negative number.

- **Solve** logarithmic equations that are more complicated by using the properties of logarithms to rewrite the equation so that it contains just one logarithm.

Properties of Logarithms

1) $\log_b M + \log_b N = \log_b (MN)$

2) $\log_b M - \log_b N = \log_b \left(\frac{M}{N}\right)$

3) $\log_b M^r = r \log_b M$

and $\log M = \log_{10} M$ **&** $\ln M = \log_e M$

Logarithmic Forms that can NOT be rewritten

$\log_b (M+N)$ **nor** $(\log_b M)(\log_b N)$

$\log_b (M-N)$ **nor** $\frac{\log_b M}{\log_b N}$ (except as a change of base)

$(\log_b M)^r$

Example 1 $2\log_3 (x+3) - \log_3 (x+1) = 3\log_3 2$

$\log_3 (x+3)^2 - \log_3 (x+1) = \log_3 2^3$ Property 3 of logarithms

$$\log_3 \left[\frac{(x+3)^2}{x+1}\right] = \log_3 8$$

$$\frac{(x+3)^2}{x+1} = 8$$

$$(x+3)^2 = 8(x+1)$$

$$x^2 + 6x + 9 = 8x + 8$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)(x-1) = 0$$

$x = 1$, and it checks.

Practice 1 $2\log(y+2) = \log(y+2) - \log 12$

Did you get $y = -\frac{23}{12}$?

Example 2 $\frac{1}{2}\log(3x+4) = \log x$

$$\log\sqrt{3x+4} = \log x$$
$$\sqrt{3x+4} = x$$
$$3x+4 = x^2$$
$$0 = x^2 - 3x - 4$$
$$0 = (x-4)(x+1)$$
$$x = 4 \text{ or } x = -1$$

We reject –1. $x = 4$ checks, and is the solution.

Practice 2 $\frac{1}{2}\log(8x-7) = \log x$

Did you get $x = 7$ or 1? Both check.

Example 3 $\ln x^2 = (\ln x)^2$

$2\ln x = (\ln x)^2$

$0 = (\ln x)^2 - 2(\ln x)$

$0 = (\ln x)(\ln x - 2)$

$\ln x = 0$ or $\ln x - 2 = 0$

$e^0 = x$ or $\ln x = 2$

$x = 1$ or $x = e^2$, and both check.

Problems:

1. $2\log_3 x = 3\log_3 5$

2. $\log_2 x - \log_2 (x+1) = 3\log_2 4$

3. $\log_{10} 5^x = \log_3 1$ (*Hint:* $\log_3 1$= ?)

4. $\frac{1}{2}\log(4x+5) = \log x$

5. $2\log_3 x - \log_3 (x-4) = 2 + \log_3 2$

6. $\ln(2x-5) - \ln(x+4) = 0$

7. $2\ln x = \ln(2x-1)$

More Challenging Problems:

8. $\log \sqrt[4]{x+1} = \frac{1}{2}$

9. $\log x^2 = (\log x)^2$

10. $\log \sqrt{x} = \sqrt{\log x}$

11. $\log(\log x) = 2$

Review: Meet with a tutor to verify your work on this worksheet and discuss some of the areas that were more challenging for you. If necessary, choose more problems from the homework to practice and discuss with the tutor.

For Tutor Use: Please check the appropriate statement:

_____ Student has completed worksheet but may need further assistance. Recommend a follow-up with the instructor.

_____ Student has mastered topic.